Reducing the complexity of logics for multiagent systems

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1. INTRODUCTION

Theories of multiagent systems (MAS), in particular those based on modal logics, often suffer from a high computational complexity. This is due in part to the combination of agents’ individual attitudes (beliefs, goals and intentions), and even more importantly to the presence of group attitudes, such as common belief and collective intention.

In teamwork, when a team of agents aims to work together in a planned and coherent way, the group as a whole needs to present a common collective attitude over and above individual attitudes of team members. Collective motivational attitudes towards a common goal are essential for achieving a sensible organization of cooperation [3, 4]. In our approach, the fundamental role of collective intention is to consolidate a group as a cooperating team, while collective commitment leads to team action, i.e., coordinated realization of individual actions by agents that have committed to do them according to the team plan (see [4]). We constructed a formal theory of teamwork from a viewpoint of the system developer who wants to reason about, specify and verify a multiagent system.

We are interested in showing how complex it is to check satisfiability of formulas with respect to our theory, and how the theories can be practically simplified in order to reduce complexity. As reminder, the satisfiability problem asks, given a formula \( \phi \), how much time and space (in terms of the length of \( \phi \)) are needed to compute whether \( \phi \) is satisfiable; i.e. whether there is a suitable Kripke model \( \mathcal{M} \) (from the class of structures corresponding to the logic) and a world \( s \in \mathcal{M} \) such that \( \mathcal{M}, s \models \phi \).

Our logics for teamwork are squarely multi-modal, in the sense that different operators are combined and may interfere [3, 4]. We have shown in [5] that individual part of our theory of teamwork is PSPACE-complete and the full system, modeling a subtle interplay between individual and group attitudes, turned out to be EXPTIME-complete, and remains so even if propositional dynamic logic is added.

It turns out that for many interesting formulas appearing in human reasoning, satisfiability tends to be easier to compute than suggested by the worst-case labels like "PSPACE-complete" and "EXPTIME-complete" [7]. Therefore it is worthwhile to investigate by which reasonable means the complexity of the teamwork theory may be reduced. As the first step of this search, inspired by [6], we explore possibilities of lowering the complexity of the satisfiability problem by restricting the modal depth of formulas or by limiting the number of propositional atoms used in the language. It turns out that bounding the depth gives a nice reduction in the individual case, but is less successful where group attitudes are concerned. Combining modal depth reduction with bounding the number of propositional atoms allows for checking the satisfiability in linear time. The jumping point of the paper is our theory of teamwork as presented in [3].

2. LOGICAL BACKGROUND

Multi-modal logics are used to formalize agents’ attitudes as well as actions they perform. In this paper, restricted to the static aspects of mental states, we only present axioms relating attitudes of agents w.r.t. propositions, reflecting states of affairs, not actions.

2.1 The Language

Definition 1. The language is based on two sets:

- a countable set \( P \) of propositional atoms;
- a finite set \( A \) of agents, denoted by numerals 1, 2, ..., n.
2.2 Semantics Based on Kripke Models

Definition 3. A Kripke model is a tuple
\[ M = (W, \{ B_i : i \in A \}, \{ G_i : i \in A \}, \{ I_i : i \in A \}, Val) \]
where \( W \) is a set of possible worlds (or states); for all \( i \in A, B_i, G_i, I_i \subseteq W \times W \) stand for the accessibility relations for each agent with respect to beliefs, goals, and intentions, respectively; and \( Val : P \times W \rightarrow \{0, 1\} \) is a valuation function that assigns the truth values to atomic propositions in worlds. A Kripke frame \( F \) is defined as a Kripke model, but without the valuation function.

Below we give truth definition for modal formulas, omitting propositional fragment (which is defined as usual). We give the definition for \( BEL \), \( E-BEL \) and \( C-BEL \) operators only. Definitions for other operators are analogous.

Definition 4. Truth definition, where \( \mathcal{M}, s \models \phi \) is read as “formula \( \phi \) is satisfied by world \( s \) in structure \( \mathcal{M} \).

1. \( \mathcal{M}, s \models \text{BEL}(i, \varphi) \) iff \( \mathcal{M}, t \models \varphi \) for all \( t \in B_i(s) \);
2. \( \mathcal{M}, s \models \text{E-BEL}(i, \varphi) \) iff for all \( i \in G, \mathcal{M}, s \models \text{BEL}(i, \varphi) \);
3. \( \mathcal{M}, s \models \text{C-BEL}(i, \varphi) \) iff \( \mathcal{M}, t \models \varphi \) for all \( i \in G, t \in (\bigcup_{i \in G} B_i) (s) \) (where \( \bigcup \) stands for transitive closure without reflexivity).

2.3 Axiom Systems for Agents’ Attitudes

Below we give a brief summary of the theory \( BGI_n \) for individual attitudes and their interdependencies as well as the teamwork theory \( BGI^{M,M}_{n} \) (fully explained in [3]).

2.3.1 Individual Motivational Operators

For goals, we take the modal system \( K_n \), for intentions the system \( KD_n \), and \( KDS_n \) for beliefs. Also, we add axioms for full goals and intentions awareness and beliefs and goals compatibility (see [3] for detailed discussion).

For each \( i \in A \):
\[ A7GB \quad \text{GOAL}(i, \varphi) \rightarrow \text{BEL}(i, \text{GOAL}(i, \varphi)) \]
\[ A8GB \quad \neg \text{GOAL}(i, \varphi) \rightarrow \neg \text{BEL}(i, \neg \text{GOAL}(i, \varphi)) \]
\[ A9IG \quad \text{INT}(i, \varphi) \rightarrow \text{GOAL}(i, \varphi) \]

Axioms \( A7GB \) and \( A8GB \) for full intentions awareness are defined analogously to the to GB-axioms. By \( BGI_n \) we denote the axiom system consisting of all axioms and rules for individual beliefs, goals and intentions as well as their interdependencies.

2.3.2 Group Beliefs

\[ C1 \quad \text{E-BEL}(i, \varphi) \rightarrow \bigwedge_{i \in G} \text{BEL}(i, \varphi) \]
\[ C2 \quad \text{C-BEL}(i, \varphi) \rightarrow \text{E-BEL}(i, \varphi \lor \text{C-BEL}(i, \varphi)) \]
\[ RC1 \quad \text{From } \varphi \rightarrow \text{E-BEL}(\psi \lor \varphi) \text{ infer } \varphi \rightarrow \text{C-BEL}(\psi) \]

2.3.3 Group Intentions

\[ M1 \quad \text{E-INT}(\varphi) \rightarrow \bigwedge_{i \in G} \text{INT}(i, \varphi) \]
\[ M2 \quad \text{M-INT}(\varphi) \rightarrow \text{E-INT}(\varphi \lor \text{M-INT}(\varphi)) \]
\[ M3 \quad \text{C-INT}(\varphi) \rightarrow \text{M-INT}(\varphi \lor \text{C-BEL}(\text{M-INT}(\varphi))) \]

By \( BGI^{M,M}_{n} \) we denote the union of \( BGI_n \) with the above axioms and rules for general and common beliefs and for general, mutual and collective intentions.

2.4 Correspondences Between Axioms and Semantics

The important feature of modal logics are correspondences between some of modal axioms and structural properties on Kripke frames. Apart from well known correspondences for systems \( KD_n \) and \( KDS_n \), there are the following correspondences for the interdependencies axioms. The semantic property corresponding to \( A7IB \) is \( \forall s,t,u((sBlt \land IItu) \rightarrow sIu) \), analogously for \( A7GB \). The property that corresponds to \( A8GB \) is
\[ \forall s,t,u((sBlt \land sBlu) \rightarrow uIIt) \],

analogously for \( A8GB \). Finally, for \( A9IG \) the corresponding semantic property is \( G, I \subseteq I \). For proofs see [4].

3. COMPLEXITY OF THE SYSTEM \( BGI^M_n \)

3.1 Effect of bounding modal depth

As was shown in [6], bounding the modal depth of formulas by a constant results in reducing the complexity of the satisfiability problem for modal logics \( KD_n \) and \( KDS_n \) to \( \text{NPTIME-} \)complete. An analogical result holds for the logic \( BGI_n \), as we shall now show.

We follow the method presented in [7], centered around the notions of a propositional tableau, a fully expanded propositional tableau (a set that along with any formula \( \psi \) contained in it, contains also all its subformulas, each of them either in positive or negated form), and a tableau. A tableau is a structure similar to a Kripke model, with nodes labelled by sets of formulas. Labels must satisfy conditions dependent on axioms of the modal system. In our case the new conditions correspond to beliefs, goals and intentions interdependencies (see [5] for details).

PROPOSITION 1. A formula \( \psi \) is \( BGI_n \) satisfiable iff there is a \( BGI_n \) tableau for \( \psi \) [5].

The following statement is crucial for the complexity result for the satisfiability problem of \( BGI_1 \) logic with bounded modal depth.

PROPOSITION 2. A formula \( \psi \) is \( BGI_n \) satisfiable iff there is a \( BGI_1 \) tableau for \( \psi \) with a size bounded by \( O(|\psi|^{\text{log}(n)}) \).

To show this we use an adjusted version of the algorithm presented in [5] and based on [7] that checks satisfiability of a \( BGI_1 \) formula by constructing a tree-like structure (called a pre-tableau) that forms the basis for a \( BGI_1 \) tableau for \( \psi \). The result follows from linear dependency between the depth of a formula and the depth of a pre-tableau needed to check its satisfiability. The main result of the section is the following theorem.

THEOREM 1. For any fixed \( k \), if the set of propositional atoms \( \mathcal{P} \) is infinite and modal depth of formulas is bounded by \( k \), then the satisfiability problem for \( BGI_n \) is \( \text{NPTIME-} \)complete.
PROOF. By proposition 2 the size of a tableau for a satisfiable formula \( \varphi \) is bounded by \( O(|\varphi|^k) \). Thus the satisfiability of the formula \( \varphi \) with bounded modal depth can be checked by the non-deterministic algorithm guessing a tableau for the satisfiable formula and checking in polynomial time if the tableau is a tableau for the formula. NPTIME-hardness follows from NPTIME-completeness of the satisfiability problem of propositional calculus.

\[ \text{THEOREM 2.} \text{ For any fixed } k, l \geq 1, \text{ if the number of propositional atoms is bounded by } l \text{ and the modal depth of formulas is bounded by } k, \text{ then the satisfiability problem for } BGI^c_{n,M} \text{ can be solved in linear time.} \]

4. COMPLEXITY OF THE SYSTEM \( BGI^c_{n,M} \)

4.1 Effect of bounding modal depth

The effect of bounding the modal depth of formulas on the complexity of the satisfiability problem for \( BGI^c_{n,M} \) logic is not as promising as in the case of \( BGI^c_{n} \). It can be shown that even if modal depth is bounded by 2, the satisfiability problem remains EXPTIME-hard.

\[ \text{THEOREM 3.} \text{ The satisfiability problem for deciding satisfiability of } BGI^c_{n,M} \text{ formulas with modal depth bounded by } 2 \text{ is EXPTIME-complete.} \]

The only difficult thing to be shown is hardness. To do this we use the method inspired by the proof of EXPTIME-hardness of the satisfiability problem for PDL given in [1, Ch. 6.8]. The method reduces the problem of deciding satisfiability of \( BGI^c_{n,M} \) formulas to the problem of deciding if there is a winning strategy for a given player in the two-person corridor tiling game described in [1, Ch. 6.8], which is EXPTIME-hard [2].

We construct a formula \( \varphi(G) \) which is satisfiable iff a particular player has a winning strategy. The constructed formula has modal depth 2 and requires either one \( K \) or one \( KD \) modality or two different \( KD45 \) modalities (i.e. at least two agents are required for \( \text{BEL} \)).

4.2 Effect of bounding the number of propositional atoms

If the number of propositional atoms is bounded by 1, the complexity of the satisfiability problem for logic \( BGI^c_{1,M} \) remains EXPTIME-hard. This can be shown by substituting propositional atoms from the proof of theorem 3, by so called \( pp \)-like formulas, that have similar properties as propositional atoms (in terms of independence of their valuations in the worlds of a model) [c.f. (6)]. A \( pp \)-like formula replacing an atom \( q_j \) is

\[ -\text{OP}(k, \neg \varphi \land \neg \text{BEL}'(1, \neg \varphi)), \]

where \( \text{OP}(k, \cdot) \) is any modal operator not used in the proof. Similarly to the case of \( BGI^c_{n} \) we have the following result.

\[ \text{THEOREM 4.} \text{ For any fixed } k, l \geq 1, \text{ if the number of propositional atoms is bounded by } l \text{ and the modal depth of formulas is bounded by } k, \text{ then the satisfiability problem for } BGI^c_{n,M} \text{ can be solved in linear time.} \]

5. DISCUSSION AND CONCLUSIONS

In this paper we analyzed the effect of bounding modal depth and the number of propositional atoms for two important components of our theory of teamwork.

For the individual fragment, both restrictions move the problem of satisfiability to the NPTIME class. For the group fragment, the results are less promising, as both techniques leave the satisfiability problem EXPTIME-hard (however the boundary of modal depth we studied in this case must be \( \geq 2 \)). Combining both techniques allows to check satisfiability in linear time in both cases. However this restriction seems to be very strong in practical applications.

In the next step we plan to check the remaining case of formulas of the group fragment of the theory with modal depth bounded by 1. Another factor we plan to study is the structure of the formulas, looking first at modal Horn formulas, which are particularly interesting from the perspective of AI applications (see [8]).

Also, when considering specific applications, it is possible to reduce some of the infinitary character of collective beliefs and intentions to more manageable proportions. Such restrictions are essential when the strongest motivational attitude, collective commitment, is considered [4] in order to produce system specifications of lower complexity.

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6. REFERENCES


