Preservation of Obligations in a Temporal and Deontic Framework

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ABSTRACT
We study logical properties that concern the preservation of future-directed obligations that have not been fulfilled yet. Our starting point is a product of temporal and deontic logics. We investigate some modifications of the semantics of the product in order to satisfy preservation properties, without losing too much of the basic properties of the product. We arrive at a semantics in which we only consider ideal histories that share the same past as the current one, and that enables a characterization of the states in which the obligations propagate. These are the states where any obligation of a formula that concerns the present moment is not violated. When there are such violations, the deontic realm switches to a lower level of ideality.

Categories and Subject Descriptors
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Theory, Legal Aspects.

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Temporal Logic, Deontic Logic, Product.

1. INTRODUCTION
A strong intuition concerning the interaction of deontic and temporal modalities is that obligations to do something in the future that are not currently met should propagate to future moments. This is particularly true for deadline obligations: if I have to finish my paper before the end of the week, and I do not finish it today, tomorrow I still have to finish it before the end of the week. But, the propagation property also pertains to future directed obligations without a deadline: if today I need to give a party someday, and I do not give the party today, then tomorrow I still have to give the party someday. Of course such properties are only valid if we assume that the deontic realm is not explicitly updated. In particular, there may be an unexpected extension of the deadline for the paper, or my friends may have organized a surprise party today, which means that I no longer have to finish the paper before the end of the week, and that I no longer have to give a party, that is, the preservation of the original obligations in the two mentioned examples is overruled. In this paper we will not be concerned with these explicit updates of the deontic realm; we only consider logical properties for the case where the obligations are settled, and where it makes sense to reason about their preservation and propagation over time on the basis of what actually happens.

An obligation to perform something in the future has a disjunctive character: either you have to do it now, next, second next, or..., etc. Now, Ross’s paradox [10] for deontic reasoning says that the disjunctive property \( Op \rightarrow O(p \lor q) \), that is valid for, for instance, standard deontic logic [11], is not intuitive under some readings. For instance, being obliged to post a letter does not imply being obliged to post or burn it. Many deontic logicians (see e.g., [7]) have argued that the paradox is due to a naive reading of the formula. If we read \( Op \) as ‘\( p \) is a necessary condition of any state that is optimal according to ones obligations’, than the property poses no problems. However, if the disjunctive information is of a temporal nature, this solution is no longer acceptable. In particular, if we accept \( Op \rightarrow O(p \lor Xp) \) as a property, we will never be able to include propagation properties like \( O(p \lor Xp) \land \neg p \rightarrow XOp \), since from the combination of those properties, we would have to accept \( Op \land \neg p \rightarrow XOp \). This property is not a desirable property under, what we think, is the most natural reading of obligation formulas in a temporal context, that is: \( Op \) means that \( p \) is obliged now, \( OXp \) means that presently we are obliged to do \( p \) next, and \( XOp \) means that next we will be obliged to do \( p \) at that next moment.

The present paper focuses on the following questions concerning the propagation problem.

Question 1: how do we formalize propagation properties?
Question 2: to what extend do we have to weaken product logics of time and obligation in order to accommodate propagation properties?

2. PRODUCT OF TEMPORAL AND DEONTIC LOGIC
We assume the reader is familiar with modal logic, possible worlds semantics, and with temporal logic.

We present here the product of Linear Temporal Logic (LTL)[9] and a version of Standard Deontic Logic (SDL)[11] with a preference relation instead of an accessibility relation for distinguishing the ideal states. This product will be the starting point for our investigation. Indeed, the product is a natural way to combine two modalities, if we want the two dimensions to have a strong interac-
of the two relations.

**DEFINITION 1 (PRODUCT FRAME, PRODUCT MODEL).** Let $T = (\mathbb{N}, <)$ and $D = (W, \leq_{\text{pref}})$ be respectively a temporal frame and a deontic frame, where $\leq_{\text{pref}}$, considered as a preference relation, is a total preorder on $W$. Then the product frame $T \times D$ is a triple $(S, L, <)$ where

- $S = \mathbb{N} \times W$ (the set of the states) is the Cartesian product of the sets $\mathbb{N}$ and $W$,
- $L = S \times S$ is the temporal relation on states such that $(i, w) < (i', w')$ if and only if $i < i'$ and $w = w'$,
- $\preff s \leq s' \in S \times S$ is the preference relation on states such that $(i, w) \preff s \leq s' (i, w')$ if and only if $w \leq_{\text{pref}} w'$ and $i = i'$.

Given a product model $T \times D$, a valuation $V$ for $T \times D \times D$ is a function $V: S \rightarrow 2^W$ that associates each state with a set of atomic propositions.

The language of this product logic has a modal operator for each of the two relations.

**DEFINITION 2 (SYNTAX OF THE PRODUCT LOGIC).** Given a countable set $P$ of atomic propositions, the temporal deontic language $T^pDL$ is defined by:

$$T^pDL = P \mid | \mid T^pDL \Rightarrow T^pDL \cup T^pDL \cup O(T^pDL)$$

**DEFINITION 3 (SATISFACTION).** A formula $\varphi$ of $T^pDL$ is interpreted on a state of a product model. Given a product model $\langle(S, L, \preff s), V\rangle$, a state $s = (i, w) \in S$, and a formula $\varphi$, we can define the satisfaction relation $\models$ by induction on $\varphi$.

Let us elaborate on the interaction of the two dimensions (deontic and temporal). For instance, there is no difference between "it is obligatory that $\varphi$ tomorrow", and "$\varphi$ will be obligatory tomorrow". This corresponds to the validity of $O(X\varphi) \leftrightarrow XO\varphi$.

The above commutativity property is characteristic for product logics. It reflects the fact the the deontic realm is not updated, as we said in the introduction. So, if it is obligatory to go to Paris tomorrow, then tomorrow it will be obligatory to go to Paris immediately, and vice versa. Now the question of the next section is whether or not we can add propagation properties to the temporal deontic product while leaving the product in tact. Intuitively, this should not be the case: propagation means that obligations are 'created' for future moments. The trigger for this creation is the circumstance that the obligations are not yet currently.

3. **ADDING A PROPAGATION PROPERTY**

As a first attempt for formalizing a propagation property to be added to the product logic, we consider:

$$O(\varphi \lor X\varphi) \land \neg \varphi \Rightarrow XO(\varphi)$$

If it is obligatory to meet $\varphi$ now, or $\varphi$ next, and $\varphi$ is not satisfied now, then next it will be obligatory to meet $\varphi$.

The first problem we have to deal with is Ross’ paradox. As argued in the introduction, what we intend to describe here is not that $O\varphi \land \neg \varphi \Rightarrow XO(\varphi)$. Yet this property would follow from 1 in combination with Ross’ property $O\varphi \rightarrow O(\varphi \lor \psi)$. To solve this problem, in the formalization of the propagation property, we explicitly exclude that $O(\varphi \lor X\psi)$ holds only because $O(\varphi)$ holds, or $O(X\psi)$ holds:

$$O(\varphi \lor X\psi) \land \neg O\varphi \land \neg O\psi \land \neg \psi \Rightarrow XO(\varphi)$$

But now we can conclude that the propagation property is not compatible with a genuine product. In fact a product model satisfies the property 2 only if it does not satisfy the hypothesis $O(\varphi \lor X\psi) \land \neg O\varphi \land \neg O\psi \land \neg \psi$. (If the hypothesis was satisfied, we could deduce $\neg XO(\psi)$ from $\neg O(\psi)$.)

The only way to preserve the propagation property is then to drop the 'no learning' property $XO\varphi \rightarrow XO\psi$. So, we will no longer have a genuine product. Obligations may now be transferred to future states. We do however preserve the 'perfect recall' property $OX\varphi \rightarrow XO\psi$ that expresses that no obligations are 'forgotten' over time.

Our goal in this section will be to define a semantics that satisfies the propagation property and the perfect recall property. To account for propagation, in the semantics we have to introduce a stronger interaction between what happens and what is obligatory, i.e., between what is true in the current world and what is true in the (next) ideal worlds. If we want to satisfy the perfect recall property we should look for a particular subset of the ideal traces. The principle of propagation then should point us to what subset to take. Our idea is that for ideal worlds at a next moment we should only take into account the worlds that share the same past as the current world.

Below, we first define the predicate $\text{SamePast}(s, s')$ which says that the states $s$ and $s'$ of a temporal deontic frame share the same past:

$$\text{SamePast}((i, w), (i', w')) \overset{\text{def}}{=} i = i' \land \forall j < i \ V(j, w) = V(j, w')$$

When interpreting an obligation in a state $s$, we only consider the states $s'$ which satisfy $\text{SamePast}(s, s')$.

**DEFINITION 4 (SEMANTICS OF THE OBLIGATION (2)).**

\[1\]We borrow this terminology from epistemic logic [5]
A problem with this definition is that if from a certain moment, no world has the same valuation as the current world, then the deontically ideal worlds shrink to the empty set. This conflicts with our desire to stay in accordance with our desire to stay in accordance with the ideal existence constraint if no world has the same valuation as the current world, then the deontic realm switches to a lower level of ideality, and the obligations of the next state do not depend on what is true now.

The propagation of obligations has been studied in the restricted case of dedicated operators for obligations with a deadline, for instance in [1, 2, 3, 4]. But, to our knowledge, the more general propagation property we have focused on is new.

We only considered a linear time setting with one agent. We cannot express “must implies can”, nor “it is obligatory to make something possible”, nor “it is obligatory for an agent to do something”. Therefore it would be interesting to further explore the link with STIT models.

Another issue is the decidability of our logic. The genuine product $LTL \times SDL$ is decidable [6] with a non elementary decision procedure, i.e., it is not far from being undecidable.

5. REFERENCES


